2023 Moab Topology Conference

The conference is funded by the National Science Foundation, Brigham Young University and Utah State University.

Monday, May 1, 2023

9:15–10:15	Ina Petkova Dartmouth College	Heegaard Floer homology methods in contact topology I
10:15–10:45	**Coffee and snacks**	
10:45–11:15	Nicholas Cazet UC Davis	Stably Irreducible Surface-Knots
11:25–12:25	Thang Le Georgia Institute of Technology	Knot invariants and algebraic structures based on knots I
12:25–1:30	**Lunch at USU Moab**	
1:30–2:30	Lisa Piccirillo Massachusetts Institute of Technology	Building closed exotic manifolds by hand I
2:30–3:00	**Coffee**	
3:00-3:30	Hugo Zhou Georgia Tech Institute of Technology	PL surfaces and genus cobordism
3:40-4:10	Agniva Roy Georgia Tech Institute of Technology	On the doubling construction for Legendrian submanifolds
4:20-4:50	Geunyoung Kim University of Georgia	On contractible $(n+3)$-manifolds
5:00–5:30	**Career panel**	

Tuesday, May 2, 2023

9:00–10:00	Ina Petkova Dartmouth College	Heegaard Floer homology methods in contact topology II
10:15–10:45	**Coffee and snacks**	
10:20–11:20	Thang Le Georgia Institute of Technology	Knot invariants and algebraic structures based on knots II
11:20–11:40	**Coffee and snacks**	
11:40–12:40	Lisa Piccirillo Massachusetts Institute of Technology	Building closed exotic manifolds by hand II
12:40–	**Lunch at USU Moab then afternoon hike**	

Wednesday, May 3, 2023

9:00–9:30	Rhea Palak Bakshi ETH Zurich	Structure of the Kauffman bracket skein module over the ring of Laurent polynomials
9:40–10:10	Katherine Merkl UC Santa Barbara	Determining which Representations of Triangle Groups Generate $SL(3, p)$
10:20–10:50	Melody Molander UC Santa Barbara	Skein Theory for Affine ADE Subfactor Planar Algebras
11:00–11:30	Rob McConkey Michigan State University	Bounds on the Cross-Cap Number (non-orientable Genus) of links
11:30	**End of conference**	

Plenary lectures

Knot invariants and algebraic structures based on knots

Thang Le Georgia Institute of Technology

Knot theory plays an important role in topology and has interesting relations to many remote branches of mathematics and physics, like number theory and non-commutative algebras. The skein module of a 3-manifold is defined using links in the 3-manifold with relations coming from identities of invariants of links in $\mathbb{R}$. When the 3-manifold is a thickened surface the skein module has a natural algebra structure, and it quantizes the character variety of the surface. We give a short introduction to the skein module/algebra theory and sketch a connection between the skein algebra theory and hyperbolic geometry/cluster algebra theory via (higher) Teichmüller theory.

Heegaard Floer homology methods in contact topology

Ina Petkova Dartmouth College

Heegaard Floer homology is a package of invariants used to study knots, links, 3-manifolds, and 4-dimensional cobordisms. With a bit more work, one also gets invariants of contact 3-manifolds, as well as of transverse and Legendrian links. The original definition of Heegaard Floer homology involves counting holomorphic curves in a high-dimensional manifold, and as a result the invariant can be hard to compute. Since, there have been various combinatorial reformulations, as well as generalizations to manifolds with boundary, which provide powerful gluing techniques for computing the Heegaard Floer invariants of closed manifolds and links. We will introduce some of these reformulations and generalizations, and discuss some recent applications to contact topology. In one direction, we will discuss spectral GRID invariants of Legendrian knots (joint work with Jubeir, Schwartz, Winkeler, and Wong); in another, an invariant of bordered contact 3-manifolds (joint work with Alishahi, Foldvari, Hendricks, Licata, and Vertesi).

Building closed exotic manifolds by hand

Lisa Piccirillo Massachusetts Institute of Technology

Historically, closed exotic 4-manifolds are built using cut and paste constructions, and their gauge theoretic invariants are computed using gluing formulae. In these talks, I'll define some new smooth 4-manifold invariants which we can compute by hand using a (Dehn) surgery formula. Armed with this I'll build explicit closed exotic 4-manifolds out of elementary handle cobordisms. We'll see some new phenomena, like closed exotic definite 4-manifolds (with fundamental group $\mathbb{Z}/2$), closed 4-manifolds with homologically essential square 0 spheres and nonvanishing invariants, and instances when knot surgery on an Alexander polynomial 1 knot can change the smooth structure. We'll try not to work too hard. This is joint work with Adam Levine and Tye Lidman.

Participant lectures

Structure of the Kauffman bracket skein module over the ring of Laurent polynomials

Rhea Palak Bakshi ETH Zurich

Skein modules were introduced by Józef H. Przytycki (and independently by Vladimir Turaev) as generalisations of the Jones and HOMFLYPT polynomial link invariants in the 3-sphere to arbitrary 3-manifolds. The Kauffman bracket skein module (KBSM) is the most extensively studied of all. However, computing the KBSM of a 3-manifold is known to be notoriously hard, especially over the ring of Laurent polynomials. With the goal of finding a definite structure of the KBSM over this ring, several conjectures and theorems were stated over the years for KBSMs. We show that some of these conjectures, and even theorems, are not true. In this talk I will briefly discuss a counterexample to Marche's generalisation of Witten's conjecture. I will show that a theorem stated by Przytycki in 1999 about the KBSM of the connected sum of two handlebodies does not hold. I will also give the exact structure of the KBSM of the connected sum of two solid tori and show that it is isomorphic to the KBSM of a genus two handlebody modulo some specific handle sliding relations. Moreover, these handle sliding relations can be written in terms of Chebyshev polynomials. This is joint work with Thang Lê and Jézef Przytycki.

Stably Irreducible Surface-Knots

Nicholas Cazet UC Davis

Are there stably irreducible surface-knots in S^4 of every (including non-orientable) genus? Livingston gave examples of stably irreducible, orientable knotted surfaces of arbitrary genus in S4. In the non-orientable case, the Kinoshita conjecture posits that all projective planes in S^4 are reducible. Yoshikawa gave infinitely many irreducible Klein bottles in S^4 where after taking connected sums give irreducible, non-orientable surfaces of arbitrary even genus. His method cannot detect if the surfaces are stably irreducible, but I will use the symmetric quandle cocycle invariant to show that there exist stably irreducible surface-knots of arbitrary even genus.

On contractible $(n+3)$-manifolds

Geunyoung Kim University of Georgia

In 1961, Mazur constructed a contractible, compact, smooth 4-manifold with boundary which is not homeomorphic to the standard 4-ball. In this talk, for any integer $n \geq 2$ we construct a contractible, compact, smooth $(n+3)$-manifold with boundary which is not homeomorphic to the standard $(n+3)$-ball.

Determining which Representations of Triangle Groups Generate $SL(3, p)$

Katherine Merkl UC Santa Barbara

In this talk, I consider some families of representation of triangle groups in $SL(n, \mathbb{Z})$ that are known to be Zariski dense in $SL(n, \mathbb{R})$. I refer to a theorem by Lubotzky that says that for almost all primes p, the images of these representations taken modulo p, will generate $SL(n, p)$. In this talk, I address the question: for which primes p, will these representations generate $SL(n, p)$?

Bounds on the Cross-Cap Number (non-orientable Genus) of links

Rob McConkey Michigan State University

The cross-cap number of a link is an invariant which considers the non-orientable spanning surfaces of the link, similar to how the genus of a link depends on the orientable spanning surfaces. In 2014 Kalfagianni and Lee found linear bounds for the cross-cap number of alternating links in relation to the coefficients of the Jones Polynomial. But what happens when we begin to look beyond alternating links? We consider a couple of families of links where such linear bounds cannot be found. Then talk about a family where we can find linear bounds for the cross cap number with respect to the twist number.

Skein Theory for Affine ADE Subfactor Planar Algebras

Melody Molander UC Santa Barbara

Subfactor planar algebras first were constructed by Vaughan Jones as a diagrammatic axiomatization of the standard invariant of a subfactor. These planar algebras also encode two other invariants of the subfactors: the index and the principal graph. The Kuperberg Program asks to find all diagrammatic presentations of subfactor planar algebras. This program has been completed for index less than 4. In this talk, I will introduce subfactor planar algebras and give some presentations of subfactor planar algebras of index 4 which have affine ADE Dynkin diagrams as their principal graphs.

On the doubling construction for Legendrian submanifolds

Agniva Roy Georgia Tech Institute of Technology

In high dimensional contact and symplectic topology, finding interesting constructions for Legendrian submanifolds is an active area of research. Further, it is desirable that the constructions lend themselves nicely to computation of invariants. The doubling construction was defined by Ekholm, which uses Lagrangian fillings of a Legendrian knot in $\mathbb{R}^{2n-1}$ to produce a Legendrian in $\mathbb{R}^{2n+1}$. Later Courte-Ekholm showed that symmetric doubles of embedded fillings are "not so interesting". In recent work the symmetric doubling construction was generalised to any contact manifold, giving two isotopic constructions related to open book decompositions of the ambient manifold. In a separate joint work with James Hughes, we explore the asymmetric doubling construction through Legendrian weaves.

PL surfaces and genus cobordism

Hugo Zhou Georgia Tech Institute of Technology

Every knot in S^3 bounds a PL disk in the four ball. But this is no longer true for knots in other three manifolds, as demonstrated first by Akbulut, who constructed a knot which does not bound any PL disk in a specific contractible four manifold. Then Levine showed that there exist knots that do not bound a PL disk in any homology four ball. What happens if we relax the condition of bounding PL disk to bounding a PL surface with some given genus? In the joint work with Hom and Stoffregen, we proved that for each n, there exists a knot K_n in an integer homology sphere that does not bound a PL surface of genus n in any homology four ball. The proof uses Heegaard Floer homology. More specifically, the obstruction comes from knot cobordism maps by Zemke and the construction uses recent filtered mapping cone formula for cables of the knot meridian.

Organizing committee

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Mark Hughes, Brigham Young University
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