**Field Axioms**

A set *F* together with two well-defined operations called addition and multiplication is a <u>field</u> if there exists elements 0, and 1 ($0 \neq 1$) and the following axioms are satisfied for all a, b, and c in *F*.

| Name | Addition | Multiplication |
|---|---|---|
| Commutativity | $a + b = b + a$ | $ab = ba$ |
| Associativity | $(a + b) + c = a + (b + c)$ | $(ab)c = a(bc)$ |
| Distributivity | $a(b + c) = ab + ac$ | $(a + b)c = ac + bc$ |
| Identity | $a + 0 = a = 0 + a$ | $a \cdot 1 = a = 1 \cdot a$ |
| Inverses | $a + (-a) = 0 = (-a) + a$ | $aa^{-1} = 1 = a^{-1}a$ if $a \neq 0$ |

A field *F* is <u>ordered</u> if, in addition to the field axioms above, there exist a relation "$\leq$" on *F* such that the following axioms are also satisfied for all a, b, and c in *F*.

i) Given *a*, and *b* in *F*, either $a \leq b$ or $b \leq a$.

ii) If $a \leq b$ and $b \leq a$ then $a = b$.

iii) If $a \leq b$ and $b \leq c$ then $a \leq c$.

iv) If $a \leq b$ then $a + c \leq b + c$.

v) If $a \leq b$ and $0 \leq c$ then $ac \leq bc$.